Permittivity and permeability in the vacuum of space revisited

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Abstract: The velocity $\boldsymbol{v}$ of a Transverse Electromagnetic (TEM) Wave in the vacuum of space is said to be the speed of light $\boldsymbol{c}$ [6]. The derivation for the velocity of the TEM wave in a vacuum is shown to include the presence of charge Q. In the pure vacuum of space, no charge or matter present, it is shown that the TEM Wave is free to propagate with velocity determined by its internal relative permittivity ε_r and permeability μ_r.

Introduction

The concepts for permittivity ε and permeability μ are introduced using Coulomb's law and the Biot-Savart law respectively. Next, the velocity of the TEM wave is shown to be related to the measured speed of light $\boldsymbol{c}$ in the vacuum of space. A unique conclusion is drawn that relates the velocity of the TEM wave to its physical internal relative permittivity and permeability in the vacuum of space free of charge.

Permittivity ε

The force $\boldsymbol{F}$ shown in figure 1 measured on charge Q_2 a distance R from the source charge Q_1 is measured using Coulomb's law (1)[1]. The unit vector $\boldsymbol{r}$ points in the direction from Q_1 to Q_2. The force $\boldsymbol{F}$ is a vector with both magnitude **F** and direction $\boldsymbol{r}$ on Q_2. The permittivity ε is determined from the medium in which the charge is situated. The permittivity in the vacuum of space ε_0 is 8.85 x 10^{-12} Fm^{-1} (Farads per meter).

$$\boldsymbol{F} = \boldsymbol{r}\ \frac{Q_1 Q_2}{4\pi\varepsilon\ \ R^2} \qquad (1)$$

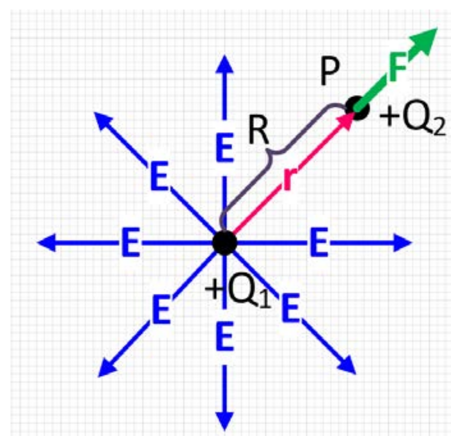

Figure 1. Point charge $\mathbf{Q_1}$ with arrows showing **E** field direction and Force on charge $\mathbf{Q_2}$ at distance **R**.

A dielectric has a dimensionless permittivity ε_r to that of a vacuum ε_0 equal to ε (2). The relative permittivity of vacuum ε_0 is 1 by definition, for paraffin 2.1 and lead glass 6 [2].

$$\varepsilon = \varepsilon_r \varepsilon_0 \qquad (2)$$

The electric field intensity **E** (3) [1] in a vacuum is defined as the force per unit charge on Q_2 at point P.

$$\boldsymbol{E} = \frac{\boldsymbol{F}}{Q_2} = \boldsymbol{r}\ \frac{Q_1}{4\pi\varepsilon_0 R^2} \qquad (3)$$

Notice that the permittivity of a vacuum ε_0 is derived in the presence of charge Q_1.

Permeability constant μ

The magnetic field **B** at a point **P** due to a current ***I*** is measured using physical parameters of the Biot-Savart Law (4)[3][4]. The current element ***I*** *dl* a distance r to point P with angle θ produces a differential magnetic field *d***B**, figure 2.

$$dB = \frac{\mu}{4\pi} \frac{I\, dl \sin\theta}{r^2} \qquad (4)$$

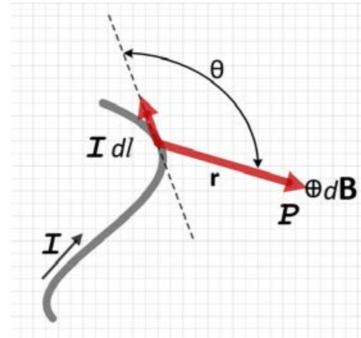

Figure 2. Biot-Savart law.

The angle **θ** is measured clockwise from the positive direction of current ***I*** along *dl* to direction of radius vector **r** extending from dl to P. The permeability μ (5) is determined by the characteristics of the medium. Magnetic materials have a dimensionless relative permeability μ_r to that of the vacuum μ_0 [5].

$$\mu = \mu_r \mu_0 \quad (5)$$

The permeability of vacuum μ_0 is $4\pi \times 10^{-7}$ Hm^{-1} (Henry per meter). The relative permeability can range from slightly less than 1 in Bismuth (0.99983) to 1,000,000 in Supermalloy [5].

Notice that the permeability of vacuum μ_0 is defined in the presence of a source current.

Electromagnetic wave equation

A Transverse Electromagnetic Wave (TEM) traveling in empty space with velocity v in the x direction with time-varying electric $\mathbf{E_y}$ and magnetic $\mathbf{B_z}$ field orientations is shown figure 3.

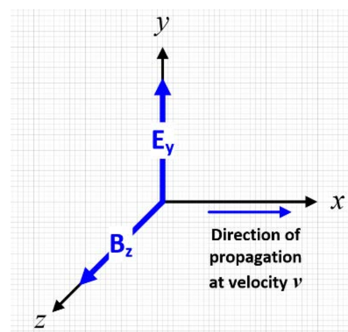

Figure 3. Field components **E** and **B** in relation to space coordinates.

The wave equation in (6)[6] relates the space and time variation of the scalar magnitude $\mathbf{E_y}$ of the electric field intensity. The velocity v of wave propagation in a lossless medium is (7)[6].

$$\frac{\partial^2 E_y}{\partial t^2} = v^2 \frac{\partial^2 E_y}{\partial x^2} \quad (6)$$

$$v^2 = \frac{1}{\varepsilon \;\; \mu} \qquad (7)$$

Per [7] the velocity of the speed of light c in the vacuum of space is (8) with the measured value of c equal to 299.79×10^6 (m/s).

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad (8)$$

Now by definition the permeability of vacuum μ_0 is $4\pi \times 10^{-7}$ Hm^{-1} combined with the measured value for the velocity of the speed of light c from (8) we get the calculated value of the permittivity in the vacuum of space ε_0 is 8.85×10^{-12} Fm^{-1} [7].

Notice again that any measurement of c in the laboratory would require the presence of matter even if the circuit for light travel is the vacuum of space.

The relative phase velocity p is defined as the ratio of the wave velocity v to the speed of light (9)[8] where μ_r is the relative permeability of the medium and ε_r is the relative permittivity of the medium.

$$p = \frac{v}{c} = \frac{\sqrt{\varepsilon_0 \mu_0}}{\sqrt{\varepsilon \quad \mu}} = \frac{1}{\sqrt{\varepsilon_r \mu_r}} \quad (9)$$

The index of refraction η in optics is defined as the reciprocal of the relative phase velocity p (10)[8].

$$\eta = \frac{1}{p} = \frac{1}{v/c} = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r} \quad (10)$$

From (10) v is (11).

$$v = \frac{c}{\sqrt{\varepsilon_r \mu_r}} = \frac{c}{\eta} \quad (11)$$

When $\varepsilon_r \mu_r$ is greater than 1 the TEM wave velocity v is less than the speed of light. Now when $\varepsilon_r \mu_r$ in (11) is 1 in a vacuum the wave velocity is equal to the speed of light.

Notice when the TEM wave is traveling with velocity v in a vacuum, this vacuum contains no matter or charge. The vacuum does contain energy in the form of a TEM wave traveling with velocity v defined in (6).

Observations

The derivation of permittivity ε_0 and permeability μ_0 for a vacuum was not performed in the absence of matter or charge Q it was derived in the presence of charge. The TEM wave velocity c in a vacuum assumes that the permittivity ε and permeability μ are equal to ε_0 and μ_0 which were not derived in a vacuum.

Conclusion

The initial or primordial TEM wave velocity v could be equal to or greater than the speed of light when the product of $\varepsilon_r \cdot \mu_r$ is ≤ 1, see (11). In (11) the relative permittivity and permeability are now associated physically with the waves internal properties since its environment in the context of this work is the vacuum of space free of charge or matter.

References

[1] Kraus J. Electromagnetics. New York, 1984, p. 12-15.
[2] Kraus, p. 57-58.
[3] Kraus, p. 152-153.
[4] Halliday D., Resnick R. Fundamentals of Physics, 1981, p. 558.
[5] Kraus, p. 215-216.
[6] Kraus, p. 381-383.
[7] Kraus, p. 387.
[8] Kraus, p. 434-435.